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### Lecture 15. A Contribution to the Empirics of Economic Growth

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1. Production function:

$$Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha}$$

2. Exogenous growth:

$$\frac{\dot{L}}{L} = n \text{ and } \frac{\dot{A}}{A} = g$$

3. Let  $k = K/(AL)$ :

$$\dot{k} = sk^\alpha - (n + g + \delta)k \quad (\text{由 } \dot{K} = sY - \delta K \text{ 推出})$$

4. Steady state of capital stock:

setting  $\dot{k} = 0$ :

$$k^* = \left[ \frac{s}{n + g + \delta} \right]^{1/(1-\alpha)}$$

5. Steady state income per capita: per worker

$$\ln \left[ \frac{Y(t)}{L(t)} \right] = \ln A(0) + gt + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta)$$

6. If factors are paid their marginal products, and since the capital income share  $\alpha$  is roughly  $1/3$ , the above equation implies an elasticity of saving of  $0.5$  and the elasticity w.r.t.  $n + g + \delta$  of  $-0.5$ . These predictions could be tested by data.

7. Assuming  $g$  and  $\delta$  are constant across countries.  $\ln A(0) + gt$  could be country specific:

$$\ln A(0) + gt = a + \varepsilon$$

8. The testable equation across countries is as follows (at time  $t$ ):

$$\ln \left[ \frac{Y}{L} \right] = a + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta) + \varepsilon$$

9. We assume that  $s$  and  $n$  are independent of  $\varepsilon$ , so we can use OLS.

Table 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.367698	1.540082	3.485333	0.0008
LNS	1.325353	0.170611	7.768281	0.0000
LNNGD	-2.013390	0.532830	-3.778672	0.0003
R-squared	0.601703	Mean dependent var		8.400780
Adjusted R-squared	0.590639	S.D. dependent var		0.952553
S.E. of regression	0.609456	Akaike info criterion		1.886678
Sum squared resid	26.74344	Schwarz criterion		1.979378
Log likelihood	-67.75042	F-statistic		54.38486
Durbin-Watson stat	1.981583	Prob(F-statistic)		0.000000

10. When we restrict the coefficients of  $\ln(s)$  and  $\ln(n + g + \delta)$  to be the same:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.085764	0.145294	48.76855	0.0000
LNS-LNNGD	1.436874	0.138818	10.35079	0.0000
R-squared	0.594757	Mean dependent var		8.400780
Adjusted R-squared	0.589206	S.D. dependent var		0.952553
S.E. of regression	0.610522	Akaike info criterion		1.877300
Sum squared resid	27.20982	Schwarz criterion		1.939100
Log likelihood	-68.39876	F-statistic		107.1389
Durbin-Watson stat	1.916935	Prob(F-statistic)		0.000000

11. Table 1 indicates:

- The coefficients of  $\ln(s)$  and  $\ln(n + g + \delta)$  have the correct signs.
- The restriction that the coefficients of  $\ln(s)$  and  $\ln(n + g + \delta)$  are equal is not rejected.
- $R^2 = 0.59$ . This says that Solow model explains a high fraction of variation in income per capita across countries.
- But the implied  $\alpha$  equals 0.59 (with a std of 0.02), much higher than 1/3. The textbook Solow model needs to be augmented but not discarded.

$\frac{\alpha}{1-\alpha} = 1.436874$

12. The Solow model augmented with human capital:

$$Y(t) = K(t)^\alpha H(t)^\beta [A(t)L(t)]^{1-\alpha-\beta}$$

13. Let  $k = K/(AL)$  and  $h = H/(AL)$ :

$$\begin{aligned}\dot{k} &= s_k k^\alpha h^\beta - (n + g + \delta)k \\ \dot{h} &= s_h k^\alpha h^\beta - (n + g + \delta)h\end{aligned}$$

where we assume that human capital, physical capital and consumption goods share the same production function and we assume that depreciate rates are the same for physical and human capital.

14. Steady state physical and human capital:

$$\begin{aligned}k^* &= \left[ \frac{s_k^{1-\beta} s_h^\beta}{n + g + \delta} \right]^{1/(1-\alpha-\beta)} \\ h^* &= \left[ \frac{s_k^\alpha s_h^{1-\alpha}}{n + g + \delta} \right]^{1/(1-\alpha-\beta)}\end{aligned}$$

15. Steady state income per capita:

$$\ln \left[ \frac{Y(t)}{L(t)} \right] = \ln A(0) + gt + \frac{\alpha}{1-\alpha-\beta} \ln(s_k) + \frac{\beta}{1-\alpha-\beta} \ln(s_h) - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta)$$

16. Similar as before, the testable equation becomes:

$$\ln \left[ \frac{Y}{L} \right] = a + \frac{\alpha}{1-\alpha-\beta} \ln(s_k) + \frac{\beta}{1-\alpha-\beta} \ln(s_h) - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta) + \varepsilon$$

17. Alternatively we can combine item 16 with item 14 and arrive at:

$$\ln \left[ \frac{Y}{L} \right] = a + \frac{\alpha}{1-\alpha} \ln(s_k) + \frac{\beta}{1-\alpha} \ln(h^*) - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + \varepsilon$$

18. The data available is a measure called SCHOOL in the paper which is a measure of  $s_h$  rather than the level  $h$ . SCHOOL is the secondary school (aged 12-17) enrollment rate multiplied by the fraction of the working-age population that is of school age (age 15-19). An imperfect measure: It ignores college education and teachers inputs etc.

Table 2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.451018	1.153417	3.858986	0.0002
LNS	0.709627	0.150343	4.720039	0.0000
LNSCH	0.728821	0.095078	7.665517	0.0000
LNNGD	-1.497194	0.402575	-3.719044	0.0004
R-squared	0.782067	Mean dependent var	8.400780	
Adjusted R-squared	0.772858	S.D. dependent var	0.952553	
S.E. of regression	0.453981	Akaike info criterion	1.310337	
Sum squared resid	14.63303	Schwarz criterion	1.433936	
Log likelihood	-45.13763	F-statistic	84.92919	
Durbin-Watson stat	2.346049	Prob(F-statistic)	0.000000	

19. The test of the restriction that the coefficients of  $\ln s_k$ ,  $\ln s_h$  and  $\ln(n + g + \delta)$  sum to zero is not rejected. The implied  $\alpha$  equals 0.29 with a std of 0.05. Thus the augmented Solow model fits the cross country data quite well.

Half life

20. Around the steady state:

多少年后能  
将差距  $\ln y^* - \ln y(t)$   
缩为初始  
差距  $\ln y^* - \ln y(0)$  的一半?

$$\frac{\dot{y}}{y} = \lambda [\ln y^* - \ln y(t)]$$

where  $\lambda = (n + g + \delta)(1 - \alpha - \beta) = 0.02$

$$\frac{d(\ln y^* - \ln y(t))}{dt} = -\frac{\dot{y}}{y} = -\lambda (\ln y^* - \ln y(t))$$

with  $\alpha = \beta = 1/3$  and  $n + g + \delta = 0.06$ , it implies a convergence speed of about 0.02.

This rate corresponds to a half life (in logarithms) of 35 years.

21. The above equation yields:

$$\ln y(t) = (1 - e^{-\lambda t}) \ln(y^*) + e^{-\lambda t} \ln y(0)$$

half life  $t$ :  $\frac{1}{2} (\ln y^* - \ln y(0)) = (\ln y^* - \ln y(0)) e^{-0.02t}$   
 $t = \frac{\ln 2}{0.02} \doteq 35$

22. Rewrite and we obtain:

$$\ln y(t) - \ln y(0) = (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln(s_k) + (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln(s_h) - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) - (1 - e^{-\lambda t}) \ln y(0)$$

which implies a negative coefficient when we regress growth rate on initial income. Endogenous Growth models (especially the AK type) predict that the coefficient is zero.

Table 3: Unconditional Convergence

Dependent Variable: LOG(YL85)-LOG(YL60)  
 Method: Least Squares  
 Date: 10/16/02 Time: 16:05  
 Sample: 1 75  
 Included observations: 75

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.568026	0.432195	1.314282	0.1929
LOG(YL60)	-0.001973	0.054743	-0.036033	0.9714
R-squared	0.000018	Mean dependent var		0.552545
Adjusted R-squared	-0.013681	S.D. dependent var		0.404225
S.E. of regression	0.406981	Akaike info criterion		1.066202
Sum squared resid	12.09123	Schwarz criterion		1.128002
Log likelihood	-37.98259	F-statistic		0.001298
Durbin-Watson stat	1.613240	Prob(F-statistic)		0.971354

23. Table 3 fails to reproduce Table III in the paper. The coefficient on initial income is  $-0.00197$  instead of  $-0.00423$ . In both tables however, this coefficient is not significant. Thus, there is no unconditional convergence.
24. When the regression controls for differences in saving and population growth rates and SCHOOL:

Table 5: Conditional Convergence

Dependent Variable: LOG(YL85)-LOG(YL60)  
 Method: Least Squares  
 Date: 10/16/02 Time: 16:13  
 Sample: 1 75  
 Included observations: 75

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.480545	0.794655	3.121537	0.0026
LOG(YL60)	-0.363743	0.066887	-5.438154	0.0000
LNS	0.546245	0.101463	5.383668	0.0000
LNNGD	-0.539011	0.286088	-1.884077	0.0637
LNSCH	0.267095	0.079721	3.350382	0.0013
R-squared	0.472140	Mean dependent var		0.552545
Adjusted R-squared	0.441976	S.D. dependent var		0.404225
S.E. of regression	0.301960	Akaike info criterion		0.507296
Sum squared resid	6.382591	Schwarz criterion		0.661796
Log likelihood	-14.02361	F-statistic		15.65272
Durbin-Watson stat	2.438641	Prob(F-statistic)		0.000000

25. Table 5 shows that the coefficient on initial income is significant and has the correct sign, consistent with the notion of conditional convergence. The implied  $\lambda$  is 0.0182 close to theoretical value of 0.02.

26. Conclusion:

- Augmented Solow model fits the data well
- No unconditional convergence. Conditional convergence is at the speed of about 1.82 percent a year, implying a half-life (in log income) of around 38 years.
- It casts doubt on the validity of endogenous growth model.