Lecture 22. The Equity Premium Puzzle

1. The Equity Premium Puzzle by Mehra and Prescott (1985) was considered one of the most influential paper in Macroeconomics. Again, it is meant to be an exercise that takes a simple dynamic general equilibrium macro model to the data. Not surprisingly, the simple model is decisively rejected by the data.

2. The puzzle attracted so much attention and led to a great deal of follow-up work. This is why Lawrence Summers in his 1991 paper gives so much credit to “The Equity Premium Puzzle”. In this sense, Mehra and Prescott (1985) is more influential than Hansen and Singleton (1982), the winner of the best paper prize (Frisch Medal) in Econometrica.


4. The rest of the presentation is taken from a note typeset by Greenwood.
\[ p = \int q(\delta')[\delta' + p']d\delta' = \int q(\delta')\{\delta' + \int q(\delta'')[\delta'' + p'']d\delta''\}d\delta' \]
\[ = \int q(\delta')\{\delta' + \int q(\delta'')\{\delta'' + \int q(\delta''')\{\delta''' + p'''\}d\delta'''\}d\delta''\}d\delta' \]
\[ = \int q(\delta')\delta'd\delta' + \int q(\delta')\int q(\delta'')\delta''d\delta''d\delta' + \int q(\delta')\int q(\delta''')\int q(\delta''')\delta'''d\delta''d\delta' + \ldots \]

Here

\[ q(\delta')q(\delta'')q(\delta''') = Q(\delta'|\delta)Q(\delta'|\delta')Q(\delta'|\delta'') \]

could be thought of as the price for a claim to one unit of consumptions three periods ahead should the event \((\delta', \delta'', \delta''')\) occur. The cost of purchasing the dividend stream three periods ahead, or \(\delta'''\), would then be \(\int \int \int Q(\delta'|\delta)Q(\delta'|\delta')Q(\delta'|\delta'')\delta'''d\delta''d\delta'\). This is the third term in the above expression.

### 4.3 The Equity Premium: A Puzzle

(Notes taken by Greenwood)

#### 4.3.1 The Problem à la Mehra and Prescott (1985)

- **Facts:**
  - From 1889-1978 the average return on equity from the Standard and Poor 500 index as 7%.
The average yield on short term debt was less than 1%.

Can such a differential be explained in a frictionless Arrow-Debreu-McKenzie economy?

- **Finding**: For the class of economies studied the average real return on equity is at a maximum 0.4 percentage points higher than on short-term debt.

- **Puzzle**: To get a low risk free interest rate in a growing economy you need a high elasticity of intertemporal substitution. To get a large equity premium, you need a high coefficient of intertemporal substitution. But one is the reciprocal of the other.

### 4.3.2 The Environment

**Tastes**

\[ U(c, \alpha) = \frac{c^{1-\alpha} - 1}{1 - \alpha}, \quad 0 < \alpha < \infty. \]

**Endowments** — $n$-state Markov chain in growth rates.

\[ y' = x'y, \]

where $x \in \{\lambda_1, ..., \lambda_n\}$ and

\[ \phi_{ij} = \Pr[x_{t+1} = \lambda_j | x_t = \lambda_i]. \]
4.3.3 Asset Pricing

\[ p_t = \beta E\{ \frac{U_1(y_{t+1})}{U_1(y_t)} [y_s + p_{t+1}] \}. \]  \hspace{1cm} (4.1)

or

\[ p_t = E\{ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{U_1(y_s)}{U_1(y_t)} y_s \}. \]

Since \( U_1(y) = y^{-\alpha} \) then

\[ p_t = P(y_t, x_t) = E[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{y_t^{\alpha}}{y_s^{\alpha}} y_s | x_t, y_t]. \]

Note that \((y_t, x_t)\) are legitimate state variables for the pricing function since \( y_s = y_t \cdot x_{t+1} \cdots x_s \). Clearly, then \( P(y, x) \) is homogeneous of degree one in \( y \). From (4.1)

\[ P(y, i) = \beta \sum_{j=1}^{n} \phi_{ij} (\lambda_j y)^{-\alpha} [y \lambda_j + P(\lambda_j y, j)] y^\alpha. \]  \hspace{1cm} (4.2)

Now, using the fact that \( P(y, i) \) is homogeneous of degree one in \( y \), conjecture a solution of the form

\[ P(y, i) = w_i y, \]

where the constant \( w_i \) will have to be determined. Substituting this solution into (4.2) yields

\[ w_i = \beta \sum_{j=1}^{n} \phi_{ij} \lambda_j^{1-\alpha} (1 + w_j), \text{ for } i = 1, 2, \cdots, n. \]  \hspace{1cm} (4.3)
Therefore,

\[ w = \beta \Lambda w + \gamma, \]

where

\[
\begin{bmatrix}
  w_1 \\
  \vdots \\
  w_n 
\end{bmatrix},
\begin{bmatrix}
  \phi_{11} \lambda_1^{1-\alpha} & \cdots & \phi_{1n} \lambda_n^{1-\alpha} \\
  \vdots & \ddots & \vdots \\
  \phi_{n1} \lambda_1^{1-\alpha} & \cdots & \phi_{nn} \lambda_n^{1-\alpha}
\end{bmatrix},
\begin{bmatrix}
  \beta \sum_j \phi_{1j} \lambda_j^{1-\alpha} \\
  \vdots \\
  \beta \sum_j \phi_{nj} \lambda_j^{1-\alpha}
\end{bmatrix},
\begin{bmatrix}
  \beta P_j \phi_{1j} \lambda_j^{1-\alpha} \\
  \vdots \\
  \beta P_j \phi_{nj} \lambda_j^{1-\alpha}
\end{bmatrix}.
\]

Thus,

\[ w = [I - \beta \Lambda]^{-1} \gamma, \]

assuming that \(|I - \beta \Lambda| \neq 0\).

What is the expected return from holding equity. The realized return, \(r_{ij}\), from moving from state \((y, i)\) to \((\lambda_j y, j)\) is

\[
r_{ij} = \frac{P(\lambda_j y, j) + \lambda_j y - P(y, i)}{P(y, i)} = \frac{\lambda_j (w_j + 1)}{w_i} - 1.
\]

**Expected Returns, Conditional on State:** The expected return on equity, conditional on that the current state is \(i\), is

\[ R_i = \sum_{j=1}^{n} \phi_{ij} r_{ij}. \]

Next consider the price of one-period discount bond in state \(i\), or \(p_t^i\). Clearly,

\[
p_t^i = P^f(c, i) = \frac{\beta E[U_1(\lambda_j y)]}{U_1(y)} = \frac{\beta \sum_{j=1}^{n} \phi_{ij} U_1(\lambda_j y)}{U_1(y)} = \beta \sum_{j=1}^{n} \phi_{ij} \lambda_j^{-\alpha}.
\]
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The return on this risk free asset is

\[ R_i^f = 1/p^f - 1. \]

**Expected Returns, Unconditional**: To calculate the expected return on either equity or bonds one needs to know the unconditional probability of being in a particular state, say \( i \). This comes from the matrix equation

\[ \pi = \pi \Phi, \]

where \( \pi = (\pi_1, ..., \pi_n) \) and \( \Phi = [\phi_{ij}] \). Therefore, the unconditional return on equity and bonds is

\[ R^e = \sum \pi_i R^e_i, \]

and

\[ R^f = \sum \pi_i R^f_i. \]

The risk premium is \( R^e - R^f \).

### 4.3.4 Findings

**Two-State Markov Chain**

\[ \lambda_1 = 1 + \mu + \delta, \]

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\[ \lambda_2 = 1 + \mu - \delta \text{ St. Dev.} \]

\[ \phi_{11} = \phi_{22} \equiv \phi \text{ and } \phi_{12} = \phi_{21} = (1 - \phi). \]

**Calibration**

For the U.S. economy the mean growth rate in consumption was 0.018. Its standard deviation and autocorrelation were 0.036 and -0.14. Matching these facts necessitated setting \( \mu = 0.018, \delta = 0.036, \text{ and } \phi = 0.43. \)

Now, clearly \( 0 < \beta < 1 \), and let \( 0 < \alpha < 10. \) Let

\[ \mathcal{X} = \{ (\alpha, \beta) : 0 < \beta < 1, 0 < \alpha < 10, \text{ and } |I - \beta \Lambda| \neq 0 \}. \]

This defines two functions, so to speak, where \( R^f = R(\alpha, \beta) \) and \( R^e - R^f = P(\alpha, \beta). \)

As can be seen the model can’t simultaneously generate an equity premium of 6.98% and risk-free return of 0.8%.

**4.3.5 Conclusions**

- Within the context of a frictionless Arrow-Debreu-McKenzie world it is difficult to rationalize why the average return on equity was so high while the risk-free return was so low.
4.4 Problems

1. A Technological Revolution: Imagine the following version of the Lucas tree economy. The economy is populated by many infinitely-lived identical agents, and equally many infinitely-lived trees. An agent’s lifetime utility is given by $\sum_{t=0}^{\infty} \beta^t U(c_t)$. A tree — the only source of production — yields a perfectly foreseen dividend, $y_t$, each period. Now, assume that the economy has been riding along in deterministic bliss. Then, unexpectedly, news arrives at $t = 0$ that a fraction $x$ of existing trees will die at the beginning of date $T$ (before dividends are paid). They will be replaced, instantaneously, by equally many new, better trees, each yielding $1 + z$ units of output, where $z > 0$. The lifetime of each tree