Lecture 8: Suboptimal Equilibrium

In the last few lectures, we present a one-sector model and we showed that the social optimal allocation can be supported by a competitive equilibrium and any competitive equilibrium is Pareto optimal, which are so-called Welfare Theorems.

The Welfare Theorems however only hold under special assumptions, such as no government intervention, no externality, no information asymmetry, no borrowing and lending constraints ...

When one of these assumptions does not hold, the Welfare Theorems are likely to break down. If a competitive equilibrium is no longer Pareto Optimal, we will call it a suboptimal equilibrium. And we will ask the question: how can a government correct the suboptimality and help the economy reach Pareto Optimal situation.

This lecture is a preparation for the forthcoming growth models with externality (Romer, JPE 1986).

A Two-Period Model with Externality

Suppose there are \(N\) individuals in this country. Each individual lives for two period. Individual \(i\) has the following utility function:

\[
\ln c_i^1 + \beta \ln c_i^2
\]

In the first period, God gives each individual \(\bar{e}\) units of goods. Each individual can decide on his own how much to consume \((c_i^1)\) and how much to save \((S_i^1)\). The saving becomes capital goods next period. In period 2, there are \(N\) firms producing consumption goods according to the technology:

\[
Y_j = F(K_j, L_j, \bar{K}) = K_j^\alpha L_j^{1-\alpha} \bar{K}^\gamma
\]

where \(\bar{K}\) is the average capital size in the country. The reason for \(\bar{K}\) to appear in an individual firm’s production function is that the higher the average size of capital stock, the more industrialized is the society and therefore the higher output an individual firm will be able to produce (positive externality).

What is going to be the Pareto Optimal allocation (Social Planner’s Problem)? What is the competitive equilibrium?

Social Planner

The existence of a Social Planner means that he can co-ordinate every individual’s behavior. For the social planner, the problem is to maximize a representative individual’s lifetime utility:

\[
\max \ln c_1 + \beta \ln c_2
\]
subject to: \( c_1 + S = \bar{e} \)
\[ S = K \]
\[ c_2 = K^\alpha L^{1-\alpha} K^\gamma \]
\[ = K^{\alpha+\gamma} L^{1-\alpha} \]

Note that the social planner understands that the size of capital stock in an individual firm is the same as the average size of capital stock because the firms have identical production functions. \( L = 1 \) because there are \( N \) individuals and each one works for one firm.

The above problem can be simplified as follows:

\[ \max \ln(\bar{e} - K) + \beta \ln(K^{\alpha+\gamma}) \]

FOC is:

\[ \frac{1}{\bar{e} - K} = \frac{\beta(\alpha + \gamma)}{K} \]

Thus

\[ K = \frac{\beta(\alpha + \gamma)}{1 + \beta(\alpha + \gamma)} \bar{e} \]

and

\[ c_1 = \frac{1}{1 + \beta(\alpha + \gamma)} \bar{e} \]

Can you explain the above formula intuitively? (An example: the lower is \( \beta \), the less patient an individual is, and therefore the higher will be his \( c_1 \))

A Competitive Equilibrium

In a competitive equilibrium, each individual thinks for himself and on his own. There is no social planner to co-ordinate their action. The market can perform the same functions of the social planner under special assumptions. When there is externality, the market will reach a suboptimal allocation instead of the optimal allocation.

To find the competitive equilibrium, let us consider the consumer’s problem, the firm’s problem and the equilibrium conditions.

The consumer’s problem

A typical consumer, \( i \), would solve the following problem:

\[ \max \ln c_1^i + \beta \ln c_2^i \]

subject to: \( c_1^i + S^i = \bar{e} \)
\[ c_2^i = (1 + r)S^i + w \]
This is equivalent to:

$$\max \ln(\bar{e} - S^i) + \beta \ln [(1 + r)S^i + w]$$

Note that the real interest rate $r$ and the real wage $w$ will be given by the market supply and demand. They are not controlled by the consumer. The only thing he controls in this problem is how much to save $S_i$.

FOC is:

$$\frac{1}{\bar{e} - S^i} = \frac{\beta(1 + r)}{(1 + r)S^i + w}$$

namely,

$$S^i = \frac{\beta(1 + r)\bar{e} - w}{(1 + \beta)(1 + r)} \quad (1)$$

The firm’s problem

The firm maximizes its profit:

$$\max K^\alpha_j L^{1-\alpha}_j \bar{K}^\gamma - K_j(1 + r) - wL_j$$

Note that any individual firm takes the average level of capital stock in the society as given $\bar{K}$. Thus $\bar{K}$ is outside of the control of any individual firm. The firm decides on how much capital to rent (because there is only one period of production, the firm returns the rental cost $rK_j$ plus the original capital $K_j$, which is assumed to have zero rate of depreciation) and how much labor to employ. In this case, the FOCs are:

$$\alpha K_j^{\alpha - 1} L_j^{1-\alpha} \bar{K}^\gamma = (1 + r) \quad (2)$$

$$\(1 - \alpha)K_j^{\alpha} L_j^{-\alpha} \bar{K}^\gamma = w \quad (3)$$

Equilibrium Conditions

In equilibrium, since everyone is identical, we must have (the side conditions):

$$S^i \equiv S^{CE}$$

$$K_j \equiv K^{CE}$$

$$\bar{K} = K^{CE}$$

$$L_j \equiv L^{CE}$$

and demand equals supply:

$$K^{CE} = S^{CE}$$

$$L^{CE} = 1$$

3
Putting all these equations and (2) and (3) to (1), we find:

\[
S^{CE} = \frac{\beta (1 + \tau) \bar{e} - w}{(1 + \beta)(1 + \tau)}
\]

\[
= \frac{\beta \alpha K_j^{\alpha - 1} L_j^{1 - \alpha} K^\gamma \bar{e} - (1 - \alpha) K_j^\alpha L_j^{-\alpha} K^\gamma}{(1 + \beta) \alpha K_j^{\alpha - 1} L_j^{1 - \alpha} K^\gamma}
\]

\[
= \frac{\beta \alpha (K^{CE})^{\alpha + \gamma - 1} \bar{e} - (1 - \alpha) (K^{CE})^{\alpha + \gamma}}{(1 + \beta) \alpha (K^{CE})^{\alpha + \gamma - 1}}
\]

\[
= \beta \frac{1}{1 + \beta} \bar{e} - (1 - \alpha) \frac{1}{(1 + \beta) \alpha} K^{CE}
\]

Thus

\[
S^{CE} = \frac{\alpha \beta}{1 + \alpha \beta} \bar{e}
\]

due to the fact that \( K^{CE} = S^{CE} \)

We can see from here that

\[
K^{CE} = \frac{\alpha \beta}{1 + \alpha \beta} \bar{e}
\]

< \( K^{P.O.} \)

Why does this happen? How can we explain this?

The fact that \( K^{CE} \) is less than \( K^{P.O.} \) is because that without co-ordination on the part of the social planner, an individual firm would underinvest. The firm’s investment generates positive externality to other firms by raising the level of average size of capital, but it does not get a direct compensation from other firms.

A related phenomenon of pollution is a case of negative externality. In the presence of negative externality of some goods, then this goods will be over-produced. That is why the environment is heavily polluted.

When there is suboptimality, government intervention can be considered. Computing a competitive equilibrium is tedious. When the CE is P.O., we can find the equilibrium by solving a social planner’s planner. When the CE is suboptimal, is there a way to compute the equilibrium by solving some kind of planning problem?

The answer is yes. The planning problem we can calculate is called a pseudo-planner’s problem. Please wait for Lecture 9.