An Endogenous Growth Model with Expanding Ranges of Consumer Goods and Producer Durables
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This paper constructs an endogenous growth model with the feature that the range of differentiated consumption goods and the range of specialized producer durables are both endogenously determined. In such a model, the domestic investment rate ought to be closely associated with the domestic saving rate even when world financial markets are integrated, provided that trade in producer durables and consumption goods is not perfect. The intuition is as follows. A country with high human capital stock will enjoy a fast rate of innovation in both producer durables and consumption goods. When trade is not perfect, the country with lower human capital stock will have to settle with slower growing ranges of producer durables and consumption goods. A fast-growing range of producer durables in a country induces a high investment rate in the country because the investment can be made in the form of new producer durables without depressing the marginal returns to capital. A fast-growing range of consumption goods in the same country induces a high saving rate because the saving can be spent on new consumption goods without reducing the marginal utility of consumption.
This resolution of the Feldstein–Horioka puzzle is rather different from previous attempts such as Obstfeld (1986), and Baxter and Crucini (1993). Obstfeld (1986) argues that saving and investment rates ought to be positively correlated because they are both increasing functions of population growth. Baxter and Crucini (1993) study how saving and investment behave in response to productivity shocks in a calibrated general equilibrium model with two countries. They find that the model generates high positive correlation between saving and investment for a large country and substantial correlation for a small country. This finding demonstrates that their model is able to explain the time series patterns on saving–investment correlations documented in Obstfeld (1986) and Tesar (1991). Their model, however, sheds no light on the original Feldstein–Horioka puzzle that a close association between long-run average domestic saving and investment rate co-exists with integrated international financial markets. In fact, it is conceivable that in their model the long-run average saving and investment rates in large and small countries could even be the same. Whereas Baxter and Crucini (1993) discuss short-run deviations from a balanced growth path, I discuss the long run properties of the balanced growth path itself.3

The model that I construct not only helps to understand the Feldstein–Horioka puzzle, but also resolves two disparity issues in growth literature. I argue that it is important to have an endogenously determined and time varying set of differentiated consumption goods as well as specialized producer durables to deal with these two issues. First, why is there persistent diversity in growth experience across countries? According to Summers and Heston (1988), per capita real GDP in Japan grew at 6 per cent per annum on average over the period from 1965 to 1980, whereas the corresponding figure in the USA was 1.7 per cent. In general, a difference of 1.5 per cent in growth rates among developed countries over several decades is commonly observed. Second, why do countries behave so differently in their saving rate? According to the World Development Report (1992), the gross domestic saving rate in 1990 was 34 per cent in Japan and 28 per cent in Germany, but only 17 per cent in the UK and 15 per cent in the USA.

The standard neoclassical growth models have difficulties in dealing with the disparity issues. They fail because of two assumptions they make.

The first assumption is that there is a single consumption good (or a fixed number of consumption goods). This assumption rules out any role for the changing variety of consumption goods to affect saving, and therefore growth. But we know that the saving behavior in an economy with an increasing range of consumption goods is different from that in an economy with a fixed range of consumption goods. Individuals who expect a high rate of introduction of new consumption goods would certainly like to save more provided that new consumption goods are not perfect substitutes for the old ones. As a remedy, this paper considers the changing variety of consumption goods explicitly.

The second assumption is that the aggregate production function takes the form, \( Y = F(K, L) \), where the marginal product of capital is decreasing in the aggregate capital stock \( K \) and approaching zero when \( K \) increases without bound. A simple

3 I thank one of the referees for pointing this out.
calculation by Lucas (1990) and a more complicated analysis by King and Rebelo (1993) show that this production function implies differences in the real interest rates across countries that are too large to be consistent with the observed slow movement of capital across borders.

The remedy I propose is to take a variant of the Ethier–Romer-type aggregate production function \( Y = H^a L^b \sum_a x_a^{1-a-b} \), where \( H \) is human capital, \( L \) is unskilled labor, \( A \) is the number of different types of producer equipment, and \( x_a \) is the quantity of type \( a \) equipment. One implication of this production function is that when comparing two closed economies, the real interest rate in the economy with a \textit{larger} aggregate capital stock does not have to be lower. The reason is that the real interest rate is not related to the aggregate capital stock \((\sum_a x_a)\), but to the marginal productivity of each type of producer equipment \((x_a)\). In fact, Romer (1990), Xie (1992), and this paper show that a richer country could have a higher real interest rate. This result is not as surprising as it appears if we treat capital and labor symmetrically: a country is rich not only because its technology can support a high wage for labor, but also because its technology can support a high return to capital.

Once the two assumptions are so modified, the model resolves the two disparity issues in a straightforward way. Countries differ in their growth performance because their endowments of human capital stock differ.\(^4\) A country with high human capital stock will have high rates of innovation in both producer durables and consumption goods. Provided that trade and knowledge spillovers across countries are not perfect, the country with lower human capital stock will not be able to take advantage of some of these innovations elsewhere, and will have to settle with slower growing ranges of producer durables and consumption goods. As a result, the country with high human capital stock will grow faster because its technology is rapidly improving. Also, the rapidly expanding range of consumption goods in this country induces its people to save more.

The rest of the paper is organized as follows. Section 2 spells out the model. Section 3 analyses a balanced growth path. Section 4 uses a numerical example to tackle the two disparity issues raised above. Section 5 studies the Feldstein–Horioka puzzle. Section 6 offers some concluding comments and suggests how one may test the theory proposed.

2. THE MODEL

2.1. Preferences. It is assumed that the long-lived representative agents have an objective function of a Dixit and Stiglitz (1977) type:

\[
\int_0^\infty \left[ \left( \int_0^V q_u^\eta \, du \right)^{(1-\sigma)/\eta} - 1 \right] \frac{1}{1-\sigma} e^{-\rho t} \, dt, \quad \rho > 0 \quad \text{and} \quad 0 < \sigma < 1,
\]

\(^4\)There are models that explain the growth differences without relying on the differences in initial conditions. See Boldrin and Rustichini (1994), Benhabib and Perli (1994), and Xie (1994) for such examples of endogenous growth models with multiple equilibria.
where $\rho$ is the rate of time preference; $\sigma$ is the inverse of the elasticity of intertemporal substitution; the parameter $\eta \in [0, 1]$ captures the degree of complementarity or substitutability among differentiated consumption goods; the letter $V$ stands for the number of varieties; and $q_v$ stands for the quantity of good $v \in [0, V]$ consumed.

For the purpose of exploring the properties of this preference structure without getting into the complication of production, I assume for now that the real interest rate is fixed and denoted by $r$. I will later show how the real interest rate is determined on a balanced growth path in equilibrium.

Let $p_v$ be the price of consumption good $v \in [0, V]$ in terms of a numeraire that will be specified later. The demand curve for the consumption goods can be derived by having the representative individual maximize the objective function (1) subject to the following constraint:

\[
\int_0^\infty \left( \int_0^V p_v q_v \, dv \right) e^{-rt} \, dt \leq W,
\]

where $W$ is the lifetime wealth.

Let $\lambda$ be the multiplier associated with constraint (2). The first-order condition is thus:

\[
\left( \int_0^V q_v^\eta \, dv \right)^{(1-\sigma-\eta)/\eta} q_v^{\eta-1} = \lambda p_v e^{-(r-\rho)t}.
\]

It will be shown that $p_v$ is constant and is the same for any $v \in [0, V]$ along a balanced growth path. Thus $q_v$ is the same across $[0, V]$. Let $q$ denote the common level of consumption. From equation (3), it is clear that

\[
\frac{(1-\sigma-\eta)}{\eta} \frac{\dot{V}}{V} - \frac{\dot{q}}{q} = -(r-\rho).
\]

Define the aggregate consumption by $C = \int_0^V q_v \, dv = Vq$. Using equation (4), we have

\[
\frac{\dot{C}}{C} = \left[ 1 + \frac{(1-\sigma-\eta)}{\eta\sigma} \right] \frac{\dot{V}}{V} + \frac{r-\rho}{\sigma}.
\]

Equation (5) contains two assertions. First, the higher the real interest rate, the faster the aggregate consumption grows. This assertion is conventional and needs no elaboration. Second, the rate of growth of variety also affects the growth rate of aggregate consumption. If different goods are independent ($\eta = 1 - \sigma$), the effect will be 100 per cent. Otherwise, the effect can be greater or less than 100 per cent, depending on whether different goods are complements or substitutes.\(^5\) Neoclassical models assume $\eta = 1$, the case of perfect substitution. In this case, the demand

\(^5\) From the demand equation (3), it is clear that different goods are complements (substitutes) if $\eta < 1 - \sigma$ ($\eta > 1 - \sigma$).
curve for $q_v$ in equation (3) is flat. Hence, no monopoly profit can be earned from being an exclusive producer of type $v$ goods. As a result, no intentional research will be done to produce new designs of consumption goods. In this paper, I argue for a necessary deviation from the neoclassical paradigm by allowing $\eta < 1$.

2.2. The Technology.

2.2.1. Research sectors $A$ and $V$. The output in research sector $A$ is designs of new producer equipment. Let $A$ be the number of designs of existing producer equipment. Let $H_A$ denote the amount of human capital engaged in research sector $A$. I assume:

$$A = \delta_A H_A A, \tag{6}$$

where $\delta_A$ is constant, depending on the choice of units for $A$ and $H_A$.

The substantive assumption embedded in the production function of new designs is that the larger the number of existing designs, the higher will be the productivity of a researcher. The functional form assumption that the production of new designs is linear in $A$ for a given level of human capital is crucial for obtaining a balanced growth path. A detailed discussion of equation (6) can be found in Romer (1990).

The output in research sector $V$ is designs of new consumption goods. Similarly, I assume:

$$V = \delta_V H_V V, \tag{7}$$

2.2.2. Manufacturing sector $Y$. Let letter $B$ stand for basic commodities such as extracted natural resources. The basic commodities appear in the production function of physical goods. The reason for introducing the basic commodities as another productive factor will be given below when I discuss the setup of the model.

For convenience, $B$ is taken as the numéraire.

Assume that the basic commodities $B$, each type of consumption good $q_v$ for $v \in [0, V']$, and each type of producer equipment $x_a$ for $a \in [0, A]$ are all produced by a common technology. This assumption is a device often used to reduce multiple production sectors to a single manufacturing sector in order to obtain a tractable one-sector model. Given this assumption, we have

$$Y = C + B + K, \tag{8}$$

where $Y$ denotes total manufacturing output, $C = \int_0^V q_v dv$ denotes the aggregate consumption as before, and $K$ denotes total producer equipment, $K = \int_0^A x_a da$.

Note that the consumption goods and basic commodities are nondurable, while all the producer equipment is perfectly durable. The consumption goods are consumed by people; the basic commodities are 'consumed' in the manufacturing sector.

The common technology used here is an extension of that used in Ethier (1982) and Romer (1990):

$$Y = H^2 \beta B \left[ \int_0^A x_a \xi da \right]^{\gamma/\xi}, \tag{9}$$
where $L$ is the physical labor services assumed to be in fixed supply; and $H_Y$ is the level of human capital employed in manufacturing. The parameter $\gamma$ is equal to $1 - \alpha - \beta - \theta$, so that the production function exhibits constant returns to scale. The parameter $\xi \in [0, 1]$ captures the degree of complementarity or substitutability among producer durables.

The total stock of human capital $H$ is taken to be fixed.

2.2.3. Discussion of the model. At this point, let me compare my setup with the ones in Grossman and Helpman (1990, 1991) and in Romer (1990). Conclusions drawn in Barro and Sala-i-Martin (1995) on related issues are also discussed when appropriate.

My model becomes similar to the variety-based growth model in Grossman and Helpman (1991) when the parameter $\delta_A$ is zero or $\xi = 1$. $\delta_A = 0$ means that research sector $A$ is not productive and $\xi = 1$ means that all producer durables are perfect substitutes. In both cases, there will be no activity in research sector $A$ and consequently no technological innovation and no growth in GDP as measured here. Thus, if the variety of consumption goods expands, the quantity of each type of consumption goods must fall over time. “This picture does not accord well with observed economic growth over long periods,” commented Barro and Sala-i-Martin (1995, p. 236), “expanding variety does occur, but this kind of growth seems to be accompanied by increases in the quantities of goods of a given type.”

My model reduces to the endogenous growth models in Romer (1990) and in Grossman and Helpman (1990) when the parameter $\delta_Y = 0$ or $\eta = 1$. In this case, the preferences conform with the neoclassical assumption and the growth rates of consumption will converge across countries when world financial markets are integrated. The reason is the following: (i) financial market integration equalizes the real interest rate $r$; and (ii) with neoclassical preferences, the growth rate of per-capita consumption is tightly related to the real interest rate, $\dot{c}/c = (r - \rho)/\sigma$. Therefore, financial market integration is sufficient to bring about equalization of growth rates of per-capita consumption.

I agree with Barro and Sala-i-Martin (1995) that, fundamentally, the only final good is the flow of utility, and the quantities of producer and consumer goods can both be thought of as intermediates that help to produce the final good. I also agree that technological progress in production is important to explain the long-run growth that we observe. What I contend in this paper is that it is premature for Barro and Sala-i-Martin to conclude that the introduction of varieties of consumer goods adds little to the understanding of the growth process. What they fail to recognize is that the expanding variety of consumption goods raises saving and therefore affects growth rates of real GDP. An endogenous growth model that allows for an expanding range of consumption goods as well as producer durables is indispensable in explaining the world economy.

If the world economy were completely integrated, the growth rates of GDP across countries would be expected to converge over time. The lack of such convergence shown in the historical data, and the observation of many existing trade barriers indicate that the degree of current world economic integration is limited. To be able to address issues of growth and trade in a world with limited trade opportunities,
I introduced a new productive factor, the basic commodities $B$. I assume that the basic commodities $B$ are the only physical goods that are traded. Essentially, the introduction of the basic commodities allows me to consider the impact of financial market integration with no actual movement of specialized capital goods and differentiated consumption goods.

2.3. Market Structure and Equilibrium Conditions. Note that consumption goods, producer durables, and the basic commodities are assumed to be produced by a common technology. This assumption implies that the prices of the various goods all equal unity if all markets are competitive. When some of the markets are not, the prices of the goods in those markets can be greater than unity. The detail of the market structure is as follows.

The market for the basic commodities $B$ is assumed to be competitive; the markets for consumption goods $v \in [0, V]$ and for producer equipment $a \in [0, A]$ are monopolistically competitive.

A research and development (R&D) firm is granted an infinitely-lived patent for an invention of either a new type of equipment or a new type of consumption good. It can organize its own production of the invented product or sell the patent to another firm, which then becomes the exclusive producer of the invented product. Assume there are a large number of firms that bid for the patent. In equilibrium, the price of the patent must equal the present value of the stream of monopoly profits that can be extracted thereafter. To calculate the monopoly profits, the demand for the new product must be derived.

First, consider the demand for a consumption good $v$. Equation (3) yields an inverse demand function for the good $v$:

$$ p_v = \frac{e^{-(r-\rho)t} \left[ \int_0^v q_v^\eta \, di \right]^{(1-\sigma-\eta)/\eta}}{\lambda} q_v^{\eta-1}. $$

(10)

The firm, which buys the patent for producing good $v$, tries to maximize its profits on each date $s \geq t$. In the inverse demand function, the variable $\lambda$—the marginal utility of wealth—is unaffected by this firm’s pricing policy because consumers spend only a tiny share of their wealth on purchases from any one firm. Also, the firm takes $\int_0^v q_v^\eta \, di$ as given because changing $q_v$ has a negligible effect on the integral.

Since at time $s$, $B(s)$ is the numeraire, and $q_v(s)$ and $B(s)$ can be converted one for one by the assumption of common technology, the marginal cost of producing one unit of $q_v(s)$ is just one. Thus, the firm’s problem at time $s \geq t$ is:

$$ \max_p p_v(s) q_v(s) - q_v(s), $$

subject to the demand curve given in equation (10). The solution is:

$$ p_v(s) = \frac{1}{\eta}, \quad \forall s \geq t. $$

(12)
The profit made at time $s$ is

$$\pi_v(s) = \left[ p_v(s) - 1 \right] q_v(s) = \frac{1 - \eta}{\eta} q_v(s).$$

Therefore the price of a patent for good $v$ at time $t$ is

$$P_v(t) = \int_t^\infty \frac{1 - \eta}{\eta} q_v(s) e^{-r(s-t)} ds.$$

Because of the symmetry among existing consumption goods, it must be true that $q_v(s) = q(s), \forall v \in [0, V]$. Hence, $P_v(t)$ is the same for any $v \in [0, V]$. For convenience, denote the common price of patents for consumption goods as $P_v(t)$,

$$P_v(t) = \int_t^\infty \frac{1 - \eta}{\eta} q(s) e^{-r(s-t)} ds.$$

Next, consider the demand for a producer equipment $a \in [0, A]$. Since a piece of equipment $a$ is durable with no depreciation, it is simpler to let the firm lease the equipment to the firms in manufacturing sector $Y$. Let $p_a$ be the rental rate of equipment $a$. For given scale of $H_Y$, $L$, and $B$, the representative firm in the manufacturing sector chooses the amount of $x_a$ of each equipment $a \in [0, A]$ to maximize its profit, taking the list of rental rates $\{p_a\}_a$ as given:

$$\text{max}_{X_a} H_Y^\alpha L^\beta B^\theta \left[ \int_0^A x_a^\xi da \right]^{\gamma/\xi} - \int_0^A p_a x_a da.$$

The inverse demand function derived from this maximization problem is

$$p_a = \gamma H_Y^\alpha L^\beta B^\theta \left[ \int_0^A x_a^\xi da \right]^{(\gamma - \xi)/\xi} x_a^{\xi-1}.$$

The firm that buys the patent for equipment $a$ takes this demand curve as given and chooses a profit maximizing rental rate $p_a$.

Since one unit of equipment $a$ can be converted from one unit of $B$, the marginal rental cost of $a$ is just the real interest rate $r$. Hence the profit-maximizing problem of the firm that produces the equipment $a$ is

$$\max_{p_a} p_a x_a - r x_a,$$

subject to the demand curve given by equation (17). The resulting monopoly rental rate $p_a = r/\xi$ is a constant mark-up over the marginal rental cost $r$.

The profit made at time $s$ by the firm holding the patent $a$ is

$$\pi_a(s) = r \left( 1 - \frac{s}{\xi} \right) x_a(s)/\xi.$$
Thus the price for the patent for equipment \( a \) is

\[
P_a(t) = \int_t^\infty \left[ r(1 - \xi) / \xi \right] x_a(s) e^{-r(s-t)} \, ds.
\]

Because of symmetry among existing producer equipment, it must be true that
\[
x_a(s) = x(s), \quad \forall a \in [0, A].
\]

Thus, I can write \( P_A(t) \) for \( P_a(t) \), the price of any patent \( a \in [0, A] \),

\[
P_A(t) = \int_t^\infty \left[ r(1 - \xi) / \xi \right] x(s) e^{-r(s-t)} \, ds.
\]

The basic commodities used in the production of \( Y \) is determined as follows:

\[
\left[ \int_0^A x_a^\xi \, da \right]^{y/\xi} = \alpha H_a^{a-1} L^\beta B^\beta - 1 \int_0^A x_a^\xi \, da \right]^{y/\xi},
\]

the unity on the left-hand side reflects the fact that \( B \) is the numeraire.

It remains to describe how the total stock of human capital \( H \) is split, in equilibrium, among \( H_A \), \( H_V \), and \( H_Y \). The split is determined by equalization of wages to human capital in all three sectors. The wages to human capital in research sectors \( A \) and \( V \) are \( P_A \delta_A A \) and \( P_V \delta_V V \), respectively. The wage to human capital in the manufacturing sector is \( \alpha H_Y^{a-1} L^\beta B^\beta - 1 \int_0^A x_a^\xi \, da \right]^{y/\xi} \). Therefore, the two equations that determine the allocation of human capital among the three sectors are:

\[
\alpha H_Y^{a-1} L^\beta B^\beta - 1 \int_0^A x_a^\xi \, da \right]^{y/\xi} = P_A \delta_A A,
\]

and

\[
P_A \delta_A A = P_V \delta_V V.
\]

3. BALANCED GROWTH PATH

Let me first give a definition of an equilibrium. An equilibrium is a collection of allocations \( \{ q^*(t), x^*(t), B^*(t), A^*(t), V^*(t), H_A^*(t), H_V^*(t), H_Y^*(t), L^*(t) \} \) and prices \( \{ p^*(t), p_A^*(t), p_B^*(t) = 1, P_A^*(t), P_V^*(t), W_H^*(t), W_L^*(t), r^*(t) \} \) that satisfy the following conditions:

(i) **Utility and Profit Maximization.**

(a) Given prices \( \{ p^*_v(t) : v \in [0, V^*(t)] \} \), the real interest rates \( \{ r^*(t) \} \), and our representative individual’s wealth \( W^* \) that is determined by the value of his patent holdings, \( \{ P_A^*(t), P_V^*(t) \} \), and the value of his human capital and unskilled labor \( \{ W_H^*(t), W_L^*(t) \} \), maximizes our representative individual’s objective function (1) subject to his budget constraint.
(b) Given the costs, \( \{ p_a^*(t) : a \in [0, A^*(t)], P_B^*(t) = 1, W_H^*(t), W_L^*(t) \} \), the quantities of inputs \( \{ x_a^*(t) : a \in [0, A^*(t)], B^*(t), H^*, L^* \} \) maximize profit for the firms in the manufacturing sector.

(c) Given the marginal rental cost of capital, \( r^*(t) \), and the demand curve for producer equipment \( a \), \( \{ p_a^*(t), x_a^*(t) \} \) maximizes the profit of the firm that produces the equipment \( a \).

(d) Given the marginal cost (unity) and the demand curve for consumption good \( v \), \( \{ p_v^*(t), q_v^*(t) \} \) maximizes the profit of the firm that produces consumption good \( v \).

(e) Given \( \{ W_H^*(t), PA^*(t) \} \), \( HA^*(t) \) maximizes the profit of the R&D firm that invents producer equipment.

(f) Given \( \{ W_H^*(t), P_v^*(t) \} \), \( H_v^*(t) \) maximizes the profit of the R&D firm that invents consumption goods.

(ii) Market Clearing.

(a) Labor market: \( L^*(t) \) equals the fixed supply of labor \( L \).

(b) Human capital market: \( H_A^*(t) + H_F^*(t) + H_M^*(t) \) equals the fixed supply of human capital \( H \).

(c) Goods market: \( C^*(t) + B^*(t) + K^*(t) = Y^*(t) \), where \( C^*(t) = \int_0^{Y^*(t)} q_v^*(t) dv \), \( K^*(t) = \int_0^{V^*(t)} x_v^*(t) da \) and \( Y^*(t) = (H_F^*(t))^\alpha (L^*(t))^\beta (B^*(t))^\gamma \). \[ \int_0^{A^*(t)} (x_a^*)^\xi da \]

(d) Patent market:

Given \( \{ r^*(s) \} \), the price of the patent for a producer equipment invented at time \( t \), \( P_A^*(t) \) equals the present value of the stream of profits that can be made subsequently by the exclusive producer of this equipment.

Given \( \{ r^*(s) \} \), the price of the patent for a consumption good invented at time \( t \), \( P_v^*(t) \) equals the present value of the stream of profits that can be made subsequently by the exclusive producer of this consumption good.

In general, it is feasible to describe this equilibrium by a set of differential equations with initial conditions. When the initial state variables take on appropriate values, the equilibrium path exhibits balanced growth. In this paper, the state variables are: \( A \), the blueprints for producer durables that have been 'discovered;' \( V \), the blueprints for consumer goods that have been 'discovered;' \( K = \int_0^A x_a da \), the stock of physical capital; and \( B \), the basic commodities that can be thought of as an essential productive input with 100 per cent depreciation. The Appendix contains a diagram showing the timing and sequencing of actions and decisions.

Along a balanced growth path the properties of some key variables are well known, for example, the constancy of the real interest rate. Since I am only interested in obtaining the long-run behavior of the economy, I use these properties to calculate the balanced growth path rather than deriving the differential equations first and then solving for the right initial conditions.

Let the real interest rate \( r \) be constant. The following calculation shows how growth rates are related to each other and to the real interest rate along a balanced growth path.

\[ ^6 \text{If the two conditions are violated, the demand and supply in the patent market will not be equal.} \]
The growth rate of $H_Y$, $H_A$, and $H_V$: Since $H$ is constant, $H_Y$, $H_A$, and $H_V$ must also be constant.

The growth rate of technological innovation $g_A$: Equation (6) indicates that

\[(25) \quad g_A = \delta_A H_A.\]

The growth rate of consumption innovation $g_V$: Equation (7) indicates that

\[(26) \quad g_V = \delta_V H_V.\]

The growth rate of each type of consumption goods $g_q$: Equation (4) indicates that

\[(27) \quad g_q = \frac{r - \rho}{\sigma} + \frac{(1 - \sigma - \eta)}{\eta \sigma} g_V.\]

The growth rate of aggregate consumption $g_C$: $g_C = g_V + g_q$.

The growth rate of the basic commodities $g_B$: Equation (22) can be rewritten as $1 = \theta Y/B$. Hence, $g_B = g_Y$.

The growth rate of each existing producer equipment $g_x$: Since the rental rate of each producer equipment $p_a = r/\xi$, the inverse demand function in equation (17) can be manipulated to give $Ax/Y = \gamma \xi/r$. Thus $g_x = g_Y - g_A$.

The growth rate of capital stock $g_K$: Since the total capital stock is the sum of all the existing producer equipment $K = \int_0^A a da = Ax$, thus $g_K = g_A + g_x$.

The growth rate of GDP $g_Y$: Since $K = Ax$ and $Ax/Y = \gamma \xi/r$, the capital–output ratio $K/Y = \gamma \xi/r$ is constant. Thus it must be true that $g_Y = g_K$.

Given the information above on growth rates, the equilibrium can be described by three independent equations in $g_A$, $g_V$, and $r$ (see the Appendix for the equations and their derivation). These equations lead to the following proposition.

**PROPOSITION 1.** If $\theta = 0$ and $\sigma > \gamma \xi$, we have:7

(i) A country with a higher stock of human capital has a higher rate of technological change and therefore a higher growth rate of GDP.

(ii) A country with a higher stock of human capital has a higher real interest rate.

**PROOF.** Similar to a proof in Xie (1992).

**REMARK.** First, the conditions $\theta = 0$, and $\sigma > \gamma \xi$ should easily be met empirically. $\theta$ has the interpretation of the share of GDP to the basic commodities and hence is close to zero. $\gamma \xi = \gamma K/Y$ is the capital income share, which is about $1/3$.

7 We also need to impose parameter restrictions $\xi = \gamma$ and $\eta = 1 - \sigma$ for the model to be well behaved. For example, if $\xi$ is very small so that the degree of complementarity among producer durables is high, there will be very rapid technological progress that makes individual's lifetime utility infinite. Also, when $\xi$ or $\eta$ approaches unity, the corresponding research sector will cease to be active.
\( \sigma \) is the inverse of intertemporal rate of substitution and is found to be large according to empirical evidence in asset pricing literature. Second, the positive correlations among \( H, g_A, \) and \( r \) stated in the proposition are intuitive. Higher \( H \) usually means more human capital can be employed in each sector, in particular the research sector \( A \). Thus, \( g_A \) is greater. The improved technology brought by more innovations therefore makes it possible to support a higher real return to capital, \( r \).

4. THE TWO DISPARITY ISSUES

I have been focusing on the analysis of a closed economy so far. Now let me use a numerical example to illustrate how the model can help resolve the two disparity issues raised in the Introduction.

The numerical example involves two countries. They have the same parameters of production and of preferences. For the benchmark case, I set \( \alpha = 0.1, \beta = 0.3, \theta = 0.1, \gamma = 0.5, \delta_A = 0.001, \delta_V = 0.00018, \rho = 0.03, \) and \( \sigma = 0.5. \) For the moment, I also set \( \xi = \gamma \) and \( \eta = 1 - \sigma \) so that consumption goods are assumed to be independent from each other, as are producer durables. I will let \( \xi \) and \( \eta \) take on different values later to discuss how substitutability and complementarity may affect the outcome and why it is important to have both differentiated consumption goods and specialized producer equipment.

The two countries differ in their size of human capital. Country 1 has 65 units and country 2 has twice as many.

When the two countries are in autarky, their long-run behavior is summarized in Table 1. The richer country has a higher real interest rate and a higher rate of growth of consumption. We see that the gap in real interest rates is a negligible 0.22%, whereas the difference in growth rates of consumption is as large as 1.41% and the difference in saving rates is over 7%.

In order to discuss the effect of financial market integration, let me assume that the two countries are now allowed to borrow and lend. The trade in consumption goods and producer equipment is still assumed to be restricted so that the only physical goods that are allowed to go across the border are the basic commodities. As I pointed out in the discussion of the model, even with this limited opportunity for trade, existing growth models with neoclassical preferences would predict that the rates of consumption growth in the two countries will converge.

<table>
<thead>
<tr>
<th>( H = 65 )</th>
<th>( H = 130 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>4.12%</td>
</tr>
<tr>
<td>( g_A )</td>
<td>2.20%</td>
</tr>
<tr>
<td>( g_e )</td>
<td>0.55%</td>
</tr>
<tr>
<td>( g_Y )</td>
<td>2.75%</td>
</tr>
<tr>
<td>( g_V )</td>
<td>0.52%</td>
</tr>
<tr>
<td>( g_q )</td>
<td>2.24%</td>
</tr>
<tr>
<td>( g_C )</td>
<td>2.75%</td>
</tr>
<tr>
<td>( S )</td>
<td>16.70%</td>
</tr>
<tr>
<td>( I )</td>
<td>16.70%</td>
</tr>
</tbody>
</table>
In contrast, the present model predicts that even if the gap in real interest rates is
removed by financial market integration or other measures, the difference in growth
rates of aggregate consumption can remain. To see this, note that when \( \eta = 1 - \sigma \),
we have \( g_C = (r - \rho) / \sigma + g_Y \). The difference in \( g_C \) can remain as long as one
country introduces new consumption goods at a faster pace than the other. In the
numerical example, a 15\% subsidy on the research sector \( A \) by the less rich country
stimulates its technological innovation, which can then support a real interest rate as
high as that in the rich country. We see from Table 2 that the differences in \( g_C \) and
\( g_Y \) between the two countries largely remain: consumption and GDP in the rich
country grow at a rate of 4.16 per cent, whereas those in the less rich country grow
at 3.15 per cent only.

As can be seen from Table 2, the difference in saving behavior also remains
significant, as the two countries have different introduction rates for new varieties of
consumption goods.

I argued intuitively in the Introduction and in the discussion of the model that it
is important to have both differentiated consumption goods and specialized pro-
ducer durables to resolve the two disparity issues. For a scientific demonstration, I
ask what happens when either \( \xi \) or \( \eta \) approaches 1 so that one of the research
sectors is not active. The experiment in the Appendix clearly illustrates that the
model with one active research sector alone loses the ability to account for the
observed diversity in growth and saving rates with integrated financial markets.

This paper, as many other studies, relies on the differences in initial conditions to
explain differences in growth and saving rates. The progress made here is that
whereas in previous studies the differences in growth and saving rates will vanish
once financial markets are integrated, in this paper they remain. This line of
literature, however, fineses the interesting and the ultimate questions as to why
countries are different in the first place and how government policy makes a
difference. These questions are partly treated in a special volume in the Journal of
Economic Theory (June, 1994) on endogenous growth models with multiple equilib-
ria. In particular, Xie (1994) speculates on whether government policy can help the
economy to pick a better equilibrium path from a continuum of equilibria.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
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<tbody>
<tr>
<td>OPEN ECONOMIES COMPARISON</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( r )</td>
</tr>
<tr>
<td>( g_A )</td>
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<tr>
<td>( g_s )</td>
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<tr>
<td>( g_Y )</td>
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<td>( g_C )</td>
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<td>( S )</td>
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</table>
5. SAVING AND INVESTMENT CORRELATION

Feldstein and Horioka (1980), using data from OECD countries for the period 1960–1974, concluded that the world capital mobility is not perfect. The empirical evidence they offer is that domestic saving is highly correlated with domestic investment. Such a high correlation, they claimed, should not be observed with perfect world capital mobility because saving in each country responds to the worldwide opportunities for investment while investment in that country is financed by the worldwide pool of capital.

Obstfeld (1986) constructed a life-cycle model for a group of small, open economies with different population growth rates. He argues that since both saving and investment rates are increasing functions of population growth rates, the high correlation between domestic savings and domestic investment is consistent with the behavior of these small countries in spite of the perfect mobility of capital. There are also discussions on the short-run correlation between domestic savings and domestic investment found in time series. See Obstfeld (1986) and Baxter and Crucini (1993).

Other explanations for the correlation between domestic investment and saving are given in Summers (1985) and Frankel (1985), among others. Summers suggests that capital is mobile internationally but that governments utilize fiscal policies to change public or private savings in such a way as to reduce the current account imbalances. Frankel argues that even though financial markets are perfectly integrated, as long as physical capital goods are not perfectly substitutable for bonds within each country, national saving rates and national investment rates can still be highly correlated if physical capital goods markets are not well integrated. In this case, since the expected real returns in physical capital are not necessarily equalized, crowding out can happen.

In order to describe the alternative explanation that the present model offers for the correlation, let me compare my explanation with that of Obstfeld (1986), Frankel (1985), and Summers (1985). Again, the two-country numerical example given above is used to help the illustration.

(i) Human capital, in contrast to the growth rate of population in Obstfeld, is the factor that drives saving and investment to move in the same direction. A country with a high stock of human capital has higher rates of innovation in both producer equipment and consumption goods. The former induces a high investment rate because expanding varieties of producer equipment increase investment opportunities. The latter induces a high saving rate because the saving can be spent in the future on new varieties of consumption goods that offer high marginal utility. This argument is supported by the numerical calculation. The saving rate and investment rate are 23.98 per cent in the rich country and much lower in the less rich country, only 16.7 per cent.

(ii) When some physical goods are nontraded, the saving–investment correlation is consistent with highly integrated financial markets that have only a tiny spread between the borrowing and the lending rate. Although the marginal productivity of nontraded capital goods can be very different, the returns to financial investment
are similar across countries. In the numerical example, some producer equipment with finite marginal productivity in the rich country may have infinite marginal productivity in the less rich one, but the difference in the real interest rates is small, only 0.22%. Thus, a 0.22% spread between the borrowing and the lending rate will keep the current accounts in balance. The 'imperfection of goods markets' argument by Frankel is formalized in this general equilibrium growth model.

(iii) When the two countries open their financial markets to each other with no spread in the borrowing and the lending rate, the country with a lower real interest rate experiences a current account surplus. If the government in that country sets the balance of its current account as a target, I suggest that it can achieve this goal by subsidizing the research sector that undertakes technological innovation. The numerical example shows that a 15% subsidy will raise its long-run equilibrium real interest rate to 4.34%, and therefore will stop the capital outflow. In this case, although the real interest rates will be 4.34% in both countries, the domestic saving rate in the less rich country will be 18.2% (see Table 2), still considerably lower than that in the rich country, 23.98%. And since there is no net borrowing and lending between the two countries in the long run, domestic saving and domestic investment rates are perfectly correlated. The viewpoint may be considered a long-run counterpart of Summers' 'policy reaction' argument (Summers, 1985).

6. CONCLUSION

Using an endogenous growth model with expanding ranges of consumer goods and of producer durables, I show that national saving rates and investment rates ought to be closely associated with each other in the long run. Countries that have high stock of human capital will have both high rates of technological innovation and high rates of introduction of new consumption goods. The former induces a high investment rate because expanding variety of producer equipment offers greater investment opportunities; the latter induces high saving rates because the saving can be spent in the future on a new variety of consumption goods without reducing the marginal utility of consumption. In other words, human capital is the factor that drives the investment and the saving rates in the same direction.

I also show that differences in growth and saving rates among developed countries can be sustained even when the differences in real interest rates are removed by world financial market integration. This result is in contrast to the common conclusion of existing growth models with neoclassical preferences that equalization of real interest rates across countries leads to equalization of growth rates of consumption.

In the analysis, the variety of consumption goods plays a vital role. With the same real interest rates, individuals in a country that has a higher rate of introducing new consumption goods have higher incentive to save and therefore enjoy higher growth rate of consumption.

Admittedly, the results in this paper hinge on the assumptions that international trade in capital goods and consumption goods does not occur and knowledge spillover in innovations does not go across the border. These are stringent assumptions and a total abolition of them will certainly lead to economic convergence in the
model. The main results, however, will continue to hold when these assumptions are partially relaxed. As long as international knowledge spillover is less than perfect (due to, for instance, language problems and trade secrets) and an important category of goods are nontradables (service-oriented technologies and 'military high-tech,' for instance), the qualitative analysis of this paper applies. For quantitative policy analysis, it surely requires effort in distinguishing tradables and non-tradables and in measuring the extent of international as opposed to domestic knowledge spillover.

To test the theoretical results presented in this paper, one needs to construct proxies for the variety of consumption goods $V$ and for the variety of producer equipment $A$. Some patent data books may be worth experimenting with in the future.

An indirect and partial test may be easier to conduct by making use of the data from developing countries. For example, if one obtains a measurement on the speed of opening consumption goods markets for a set of developing countries, one can test whether there is the correlation suggested here between the process of trade liberalization and the saving rate.

**APPENDIX**

A1. **Characterization of the Balanced Growth Path.** I will show that the balanced growth path can be characterized by three equations on three variables, $g_A, g_V, \text{ and } r$.

Since $g_Y = g_B$, the production function yields

\[
g_Y = \frac{1}{1 - \theta} \left[ \frac{\gamma}{\xi} g_A + \gamma g_x \right].
\]

Also, we know that $g_Y = g_K = g_A + g_x$, which can be substituted into the equation above, and we obtain

\[
g_x = \frac{[\gamma - \xi(1 - \theta)]}{(\alpha + \beta)\xi} g_A.
\]

Thus, the price of a patent for any producer equipment is

\[
P_A(t) = \frac{r(1 - \xi) x(t)}{\xi r - [\gamma - \xi(1 - \theta)] g_A/(\alpha + \beta)}.
\]

The price of a patent for any consumption good is

\[
P_Y(t) = \frac{1 - \eta}{\eta} \frac{q(t)}{r - [(r - \rho)/\sigma + (1 - \sigma - \eta) g_Y/(\eta \sigma)]}.
\]
Substitute equations (30) and (31) into wages equalization equations (23) and (24). Equation (23) implies

\[ \Lambda \left( r - \frac{[\gamma - \xi(1 - \theta)]}{(\alpha + \beta)\xi} g_A \right) = \delta_A H_Y, \]

where \( \Lambda = \alpha / [\gamma(1 - \xi)] \). Note that \( H_Y = H - H_A - H_V = H - (g_A / \delta_A) - (g_V / \delta_V) \), equation (32) can be rewritten as

\[ (1 - \Lambda \frac{[\gamma - \xi(1 - \theta)]}{(\alpha + \beta)\xi}) g_A + \left( \frac{\delta_A}{\delta_V} \right) g_V + \Lambda r - \delta_A H = 0. \]

Equation (24) implies that \( g_A + g_x = g_V + g_q \), that is

\[ \frac{(1 - \xi)\gamma}{(\alpha + \beta)\xi} g_A - \left( 1 + \frac{1 - \sigma - \eta}{\eta\sigma} \right) g_V - \frac{r}{\sigma} + \frac{\rho}{\sigma} = 0. \]

Also, (24) implies that the consumption-capital ratio must satisfy the following

\[ \frac{C}{K} = \Gamma \frac{r - \left[ [(r - \rho) / \sigma + (1 - \sigma - \eta) g_V / (\eta\sigma)] \right]}{r - \left[ \gamma - \xi(1 - \theta) \right] g_A / \left[ (\alpha + \beta)\xi \right]}, \]

where \( \Gamma = [(1 - \xi)\delta_A\eta] / [\xi\delta_V(1 - \eta)] \) is constant.

The capital accumulation equation can be rewritten as

\[ g_K + \frac{C}{K} - \left( \frac{Y}{K} - \frac{B}{K} \right) = 0. \]

Substituting (35) into (36) and using equations (17) and (22) yield

\[ \frac{(1 - \xi)\gamma}{(\alpha + \beta)\xi} g_A + \Gamma \frac{r - \left[ [(r - \rho) / \sigma + (1 - \sigma - \eta) g_V / (\eta\sigma)] \right]}{r - \left[ \gamma - \xi(1 - \theta) \right] g_A / \left[ (\alpha + \beta)\xi \right]} \]

\[ - \frac{r(1 - \theta)}{\gamma\xi} = 0. \]

Equations (33), (34), and (37) are the three equations that describe the balanced growth path.

A2. The Timing of Events. For convenience, the timing and sequencing of actions and decisions are described here in discrete time framework.
Sector $A$ uses human capital to produce new designs of producer durables: $A(t + 1) - A(t) = \delta_A H_A(t) A(t)$

Sector $V$ uses human capital to produce new designs of consumer goods: $V(t + 1) - V(t) = \delta_V H_V(t) V(t)$

| $A(t)$ | $A(t + 1)$ |
| $V(t)$ | $V(t + 1)$ |
| $K(t)$ | $K(t + 1)$ |
| $B(t)$ | $B(t + 1)$ |

Manufacturing sector uses human capital, labor, basic commodities and producer durables to produce goods $Y$:

$$Y(t) = H^\xi(t) L^\beta B^\theta(t) \left[ \int_0^{A(t)} x^\xi_a(t) \, da \right]^{\gamma / \xi}$$

A firm who got the patent of type $a \in [0, A(t + 1)]$ equipment converts $x_a(t + 1) - x_a(t)$ units of $Y$ to type $a$ equipment. The firm needs to pay a marginal rental cost $r$. Note that $x_a(t) = 0$ for $a \in (A(t), A(t + 1)]$. Each unit of the specialized equipment is rented to the manufacturing sector at $r / \xi$ at time $t + 1$.

A firm who got the patent of type $v \in [0, V(t)]$ consumption goods converts $q_v(t)$ units of $Y$ to type $v$ consumption goods. For that purpose, the firm needs to pay a marginal cost $1$. The firm sells the type $v$ good to the consumers at the price $1 / \eta$.

Competitive firms convert $B(t + 1)$ units of $Y$ into basic commodities.

To summarize, we have $Y(t) = K(t + 1) - K(t) + C(t) + B(t + 1)$, the counterpart of equation (8) (Note that $K(t) = \int_0^{A(t)} x_a(t) \, da$).

**A3. Importance of the Variety of Consumption Goods.** I would like to show that it is crucial to have the variety of consumption goods.

Note that different consumption goods are complements (substitutes) when $\eta$ is less (greater) than $1 - \sigma$. When $\eta$ approaches 1, all consumption goods become perfect substitutes and thus no consumption goods will be invented. The task here is to show that when $\eta$ increases and approaches 1, the ability of the model to resolve the two disparity issues diminishes.

Figure 1 depicts the growth rates of GDP for the two countries when the two are in autarky.
From this diagram alone, one might think, and Barro and Sala-i-Martin (1995) seem to argue, that even when $\eta$ approaches 1, the diversity in growth rates still exists provided that the two countries have different sizes of human capital. However, when we look at the saving behavior in Figure 2, we realize that as $\eta$ approaches 1, the ability of the model to explain the difference in savings rate weakens.

If we go further and examine the real interest rates in Figure 3, we find that the differences in growth rates in Figure 1 is more and more accounted for by the differences in the real interest rates as $\eta$ increases. As a result, when the real interest rate differentials are removed by financial market integration or other policy measures, the growth differential will disappear accordingly.
A4. Importance of the Variety of Producer Durables. I would like to show that having the variety of consumption goods alone is not sufficient to explain the observed facts.

Note that when $\xi$ increases and approaches 1, different producer durables become more closely substitutable. This eventually kills the incentive for innovation in producer durables and leads to zero real growth in GDP. Figures 4 and 5 show that as $\xi$ approaches 1, the differences in growth and savings across countries shrink to zero.

Figure 6 shows the real interest rates. As $\xi$ approaches 1, the manufacturing sector and sector $V$ will split the human capital with no activity in sector $A$. The rich country therefore naturally has greater variety of consumption goods. Because
of zero real growth of GDP, we must have \((r - \rho)/\sigma + g_Y = 0\). Thus the rich country has lower real interest rate in this case (see the footnote on Proposition 1).

Thus, I conclude that both sector \(A\) and \(V\) should be sufficiently active in order to explain the observed patterns of growth and saving in the world.

REFERENCES


